AALIM MUHAMMED SALEGH COLLEGE OF ENGINEERING

Department of Mathematics

Model Examination

IV Semester B.E. (ECE)

MA2261 –PROBABILITY AND RANDOM PROCESSES

Time : 3 hours Max.Marks:100

Part A (10 X 2=10 Marks)

Answer ALL questions

1. Let X be a random variable taking values –1, 0 and 1 such that P(X= -1)= 2 P(X= 0)=P(X=1). Find the mean of 2 X –5.
2. If M.G.F. of a random variable X is $M_X(t) = (0.3\,e^t + 0.7)^{10}$ .What is the M.G.F. of Y=3X+2.? Also find E (Y).
3. Find K, if the joint probability density function of a bivariate random variable (X,Y) is given by f(x,y)= $\begin{cases} k(1-x)(1-y) \text{ if } 0 < x < y < 1 \\ 0, \text{ otherwise} \end{cases}$
4. Show that the correlation co-efficient r lies between -1 and 1.
5. When is a random process is said to be mean ergodic?
6. Prove that the sum of two independent poisson process is a poisson process.
7. The auto correlation function $R_{XX}(\tau) = \dfrac{25\tau^2+36}{6.25\tau^2+4}$ Find mean and variance of the process.
8. Define spectral density and cross-spectral density.
9. Find the system transfer function, if a Linear Time Invariant system has an impulse function $H(t) = \begin{cases} \frac{1}{2c}, |t| \le c \\ 0, |t| > c \end{cases}$
10. Define White Noise Process.

PART B (5 x 16 = 80 marks)

11.a.(i) A random variable X has the following probability distribution

X	0	1	2	3	4	5	6	7
P(x)	0	a	2a	2a	3a	a^2	$2a^2$	$7a^2+a$

Find (i) P(0<X<5) (ii) P(1.5<X<4.5/X>2) and (iii) the smallest value of n for which P(X≤n)>1/2.

(ii) . Let X be a R.V with p.d.f f(x)= $\begin{cases} \frac{1}{3}e^{\frac{-x}{3}} & \text{for } x \ge 0 \\ 0, \text{otherwise} \end{cases}$

Find (i) P(X > 3) (ii) Moment generating function of X (iii) E(X) & Var (X).

(OR)

b (i) Define normal distribution. Find the moment generating function, mean, and variance. (ii) The length of time a person speaks over phone follows exponential distribution with mean 1/6 What is the probability that the person will talk for (a)more than 8 minutes (b) between 4 and 8 minutes?

12. a (i) The joint p.d.f. of the two of the two dimensional r.v. *(X,Y)* is given by

f(x,y)= $\begin{cases} \frac{8xy}{9}, & for\ \ 1 < x < y < 2. \\ 0, & otherwise \end{cases}$ Find the marginal density functions of X and Y.

Find also the conditional density functions *f (x/y)* and *f (y/x)*

(ii) The joint p.d.f of a two dimensional random variable (X,Y) is given by

$$f(x,y)=\begin{cases} 4xye^{-(x^2+y^2)} & x \geq 0, y \geq 0 \\ 0, \text{ otherwise} \end{cases}$$ Find the density function of $U = \sqrt{x^2 + y^2}$

(OR)

b (i) If the joint p.d.f. of *X* and *Y* is given by $f(x,y)=\begin{cases} \frac{(x+y)}{3}, & \text{for } 0 < x < 1, \quad 0 < y < 2 \\ 0, & \text{otherwise} \end{cases}$.

Obtain the regression curves of *Y* on *X* and of *X* on *Y*.

(ii) The life time of a certain brand of an electric bulb may be considered a random variable with mean 1200 hours and a S.D 250 hours. Find the probability using CLT that the average life time of 60 bulbs exceeds 1250 hours

13. a (i) Discuss the stationarity of the random process $X(t) = 10\cos(wt + \theta)$ if A and w are constants and θ is a uniformly distributed random variable in $(0, 2\pi)$.

(ii) If {X(t)} is a WSS process with autocorrelation $R(t) = Ae^{-a|\tau|}$ Determine the second order moment of the RV {X(8) - X(5)}.

(OR)

b (i) If the process $\{X(t); t \geq 0\}$ is a Poisson process with parameter λ, obtain P[X(t) = n]. Is the process first order stationary ?

(ii) The tpm of the markov chain $\{X_n\}$,n=1,2,3,… having states 1,2,3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is $P^{(0)} = (0.7 \;\; 0.2 \;\; 0.1)$ Find (i)$P(X_2=3)$ (ii)$P(X_3=2, X_2=3, X_1=3, X_0=2)$.

14 a (i) The auto correlation function of the random binary transmission X(t) is given by

$$R(\tau) = \begin{cases} 1 - \frac{|\tau|}{T}, & \text{for } |\tau| < T \\ 0 & \text{for } |\tau| < T \end{cases}$$ Find the power spectrum of the process.

(ii) Let X(t) and Y (t) be both zero-mean and WSS random processes Consider the random process Z(t) defined by Z(t) = X(t) + Y (t). Find

(1) The Auto correlation function and the power spectrum of Z(t) if X(t) and Y (t) are jointly WSS. (2) The power spectrum of Z(t) if X(t) and Y (t) are orthogonal.

(OR)

b (i) State and prove Weiner - Khintchine Theorem.

(ii) The cross-power spectrum of real random processes {X(t)} and {Y(t)} is given by

$$S_{xy}(\omega)= \begin{cases} 1 + j\omega, \text{for } |\omega| < 1 \\ 0, \text{elsewhere} \end{cases}$$ .Find the cross -correlation function .

15. a (i) Prove that if the input to a time-invariant stable linear system is a WSS process then the output will also be WSS process.

(ii) State and prove the fundamental theorem on the power spectrum of output of a linear system. i.e.,With usual notation S.T $S_{yy}(w) = S_{xx}(w)\,|H(w)|^2$ where $S_{yy}(w)$ & $S_{xx}(w)$ are the spectral density function of Y(t) and X(t) and H(w) is the system transfer function.

(OR)

b (i) If N(t) is a band limited white noise centered at a carrier frequency ω_0 such that

$$S_{NN}(\omega)=\begin{cases} \frac{N_0}{2}, & |\omega - \omega_0| < \omega_B \\ 0, \text{otherwise} \end{cases}$$ find the auto correlation of the process{N(t)}.

(ii) If {X(t)} is a band limited processes such that $S_{xx}(\omega)=0$ when $|\omega|>\sigma$, prove that $2[R_{XX}(0)-R_{XX}(\tau)] \leq \sigma^2\tau^2 R_{xx}(0)$.