AALIM MUHAMMED SALEGH COLLEGE OF ENGINEERING

Department of Mathematics

MA2261- PROBABILITY & RANDOM PROCESSES

MODEL EXAM

(IV – SEMESTER/II- ECE)

Time : 3.00 hour Max Marks: 100

(Answer ALL the Questions)

PART-A (10x 2 =20 marks)

1. The CDF of a continuous random variable is given by F(x)= $\begin{cases} 0, x < 0 \\ 1 - e^{-x/5}, x \geq 0 \end{cases}$ .Find the pdf and mean of X.
2. If X is a normal random variable with mean zero and variance σ^2 , Find the PDF of Y=e^X.
3. Let X and Y be continuous random variables with joint probability density function $f_{xy}(x,y)$= x(x-y)/8 , 0 < x < 2 , - x< y <x and $f_{xy}(x,y)$= o elsewhere. Find $f_{y/x}(y/x)$.
4. The regression equation are 3x+2y=26 and 6x+y=31.Find the correlation coefficient between X and Y.
5. When is a random process said to be mean ergodic?
6. If {X(t)} is a Gaussian process with μ(t)=3 and $c(t_1,t_2)=4e^{-|t_1-t_2|}$. Find the variance of X(10) - X(6).
7. The autocorrelation function of a stationary random process is $R_{XX}(\tau)=25+\frac{4}{1+6\tau^2}$. Find the mean and variance of the process.
8. Prove that for a WSS process {X(t)}, $R_{XX}(t,t+\tau)$ is an even function of τ.
9. Write the cross correlation of input and output.
10. Define white noise.

PART-B (5 x 16 = 80 marks)

11. (a) (i) The probability function of an infinite discrete distribution is given by P(X=j)= $\frac{1}{2^j}$, j= 1, 2,3….., find mgf , mean of X,P(X is even),P(x is divisible by 3).

(ii) A continuous R.V.X has the pdf f(x) = $\begin{cases} \frac{k}{1+x^2}, -\infty < x < \infty \\ 0, otherwise \end{cases}$ find (1) the value of K (2)Distribution function of X (3) P(X $\geq$ 0).

(OR)

(b) (i)Derive the mgf of Poisson distribution and hence or otherwise deduce its mean and variance.

(ii) The marks obtained by a number of students in a certain subjects are approximately normally distributed with mean 65 and standard deviation 5. If 3 students are selected at random from this group, what is the probability that atleast one of them would have scored above 75?

12. (a) (i) The joint probability density function of two dimensional random variable (X,Y) is f(x,y) = $\begin{cases} 2-x-y, 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, \quad otherwise \end{cases}$. Find the correlation coefficient between X and Y.

(ii) The regression equation of X and Y is 3Y-5X+108=0. If the mean value of Y is 44 and the variance of X is 9/16th of the variance of Y.Find the mean value of X and the correlation coefficient.

(OR)

(b) (i) The life time of a particular variety of electric bulb may be considered as a random variable with mean 1200 hours and standard deviation 250 hours. Using CLT, find the probability that the average life time of 60 bulbs exceeds 1250 hours.

(ii) X and Y are independent with a common PDF(exponential) f(x) = $\begin{cases} e^{-x}, x \geq 0 \\ 0, x < 0 \end{cases}$ and f(y) = $\begin{cases} e^{-y}, y \geq 0 \\ 0, y < 0 \end{cases}$ Find the PDF for X-Y.

13(a) (i) State the postulates of a Poisson processes and derive the probability distribution. Also prove that the sum of two independent Poisson processes is a Poisson process.

(ii) The process {X(t)} whose probability distribution under certain condition is given by

P[X(t)=n] = $\frac{(at)^{n-1}}{(1+at)^{n+1}}, n = 1,2, \ldots and \ \frac{at}{1+at}, n = 0$.Is {X(t)} is first order stationary.

(OR)

(b) (i) A random process X(t) defined by X(t)= A cost + B sint, where A and B are independent random variables each of which takes a value -2 withh probability 1/3 and a value 1 with probability 2/3. Show that X(t) is wide-sense stationary.

(ii) A man either drives a car or catches a train to go to office each day. he never goes 2 days in a row by train but if he drives one day then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if 6 appeared. Find (i) the probability that he takes a train on the third day (ii) the probability that he drives to work in the long run.

14 (a) (i)The Auto correlation function of WSS process is given by R(τ)=$a^2 e^{-2\alpha|\tau|}$ determine the power spectral density of the process.

(ii) Given the power spectral density of a continuous process as $(\omega) = \frac{\omega^2+9}{\omega^4+5\omega^2+4}$. Find the mean square value of the process.

(OR)

(b)(i) State and prove Weiner-Khintchine Theorem.

(ii) The cross-power spectrum of real random processes {X(t)} and {Y(t)} is given by

$S_{xy}(\omega) = \begin{cases} a + bj\omega, for \ |\omega| < 1 \\ 0, \ elsewhere \end{cases}$. Find the cross correlation function.

15(a) (i) If the input to a time invariant, stable, linear system is a WSS process, prove that the output will also be a WSS process.

(ii) Let X(t) be a WSS process which is the input to a linear time invariant system with unit impulse h(t) and output Y(t), then prove that $S_{yy}(\omega) = |H(\omega)|^2 S_{xx}(\omega)$.

(OR)

(b) (i) If {N(t)} is a band limited white noise centered at a carrier frequency ω_0 such that

$S_{NN}(\omega) = \begin{cases} \frac{N_0}{2}, & for \ |\omega - \omega_0| < \omega_B \\ 0, \ elsewhere \end{cases}$. Find the autocorrelation of {N(t)}.

(ii) Consider a system with transfer function $\frac{1}{1+j\omega}$. An input signal with autocorrelation function $m\delta(\tau) + m^2$ is fed as input to the system. Find the mean and mean- square of the output.