

AALIM MUHAMMED SALEGH COLLEGE OF ENGINEERING

Department of Mathematics

Model Examination

IV Semester B.E. (ECE)

MA2261 –PROBABILITY AND RANDOM PROCESSES

Time : 3 hours

Max.Marks:100

Part A (10 X 2=10 Marks)

Answer ALL questions

1. Find the moment generating function of binomial distribution.
2. A continuous random variable X has probability density function $f(x) = \begin{cases} 3x^2, & 0 \leq x \leq 1 \\ 0, & \text{other wise} \end{cases}$ Find k such that $P(X > k) = 0.5$
3. Find the acute angle between the two lines of regression, assuming the two lines of regression.
4. State Central Limit Theorem for iid random variables.
5. Define a wide sense stationary process.
6. State the postulates of a Poisson process.
7. Prove that $S_{xy}(\omega) = S_{yx}(-\omega)$
8. State any two properties of cross correlation function.
9. Define a linear time - invariant system.
10. Define Band-Limited white noise.

PART-B (5 x 16 = 80 marks)

11. a(i) The probability density function of a random variable X is given by $f(x) = \begin{cases} x, & 0 < x < 1 \\ k(2-x), & 1 < x < 2 \\ 0, & \text{otherwise} \end{cases}$
 - (1) Find the value of k
 - (2) Find $P(0.2 < x < 1.2)$
 - (3) What is $P[0.5 < x < 1.5/x = 1]$
 - (4) Find the distribution function of f(x).

(ii) Describe gamma distribution. Obtain its moment generating function. Hence compute its mean and variance.

OR

- b(i) An office has four phone lines. Each is busy about 10% of the time. Assume that the phone lines act of independently.
 - (1) What is the probability that all four phones are busy ?
 - (2) What is the probability that atleast two of them are busy ?
 - (ii) If X is uniformly distributed in (-1, 1), then find the probability density function of $Y = \sin \frac{\pi X}{2}$
12. a(i) The joint pdf of a two-dimensional random variable (X,Y) is given by $f(x,y) = \begin{cases} xy^2 + \frac{x^2}{8}; & 0 \leq x \leq 2, 0 \leq y \leq 1 \\ 0 & ; \text{otherwise} \end{cases}$ Compute (i) $P(Y < \frac{1}{2})$ (ii) $P(X > 1/Y < \frac{1}{2})$ (iii) $P(X + Y \leq 1)$
 - (ii) Let $X_1, X_2, X_3, \dots, X_n$ are uniform variates with mean =2.5 and variance = 3/4, use CLT to estimate $P(108 \leq S_n \leq 126)$ where $S_n = X_1 + X_2 + X_3 + \dots + X_n$ n = 48.

OR

- b(i) Let (X Y ,) be a two dimensional random variable and the probability density function be given by $f(x,y) = x+ y, 0 \leq x, y \leq 1$, Find the p.d.f of $U = XY$
 - (ii) The regression equation of X on Y is $3Y - 5X + 108 = 0$. If the mean value of Y is 44 and the variance of X is 9/16th of the variance of Y . Find the mean value of X and the correlation coefficient.
13. a(i) Define a semi random telegraph signal process and prove that it is evolutionary

(ii) A random process has sample functions of the form $X(t) = A \cos(\omega t + \theta)$ where ω is constant, A is a random variable with mean zero and variance one and θ is a random variable that is uniformly distributed between 0 and 2π . Assume that the random variables A and θ are independent. Is $X(t)$ a mean - ergodic process ?

OR

b(i) If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2 per minute, find the probability that the interval between 2 consecutive arrivals is

(1) more than 1 minute (2) between 1 minute and 2 minutes and (3) 4 min. or less.

(ii) Prove that the interval between two successive occurrences of a Poisson process with parameter λ has an exponential distribution with mean $1/\lambda$.

(iii) Prove that a random telegraph signal process $Y(t) = \alpha X(t)$ is a Wide Sense Stationary Process when α is a random variable which is independent of $X(t)$, assume value -1 and $+1$ with equal probability and

$$R_{XX}(t_1, t_2) = e^{-2\lambda|t_1 - t_2|}$$

14. a(i) Find the autocorrelation function of the periodic time function of the period time function $\{X(t)\} = A \sin \omega t$

(ii) Find the power spectral density of the random process whose auto correlation function is

$$R(\tau) = \begin{cases} 1 - |\tau|, & \text{for } |\tau| \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

OR

b(i) The cross-power spectrum of real random processes $\{X(t)\}$ and $\{Y(t)\}$ is given by

$$S_{xy}(\omega) = \begin{cases} a + \frac{ib\omega}{\alpha}, & \text{for } -\alpha < \omega < \alpha \text{ and } \alpha > 0 \\ 0, & \text{elsewhere} \end{cases}$$

function.

(ii) If $\{X(t)\}$ and $\{Y(t)\}$ are two random processes with auto correlation function $R_{XX}(\tau)$ and $R_{YY}(\tau)$ respectively

then prove that $|R_{XY}(\tau)| \leq \sqrt{R_{XX}(0)R_{YY}(0)}$ Establish any two properties of auto correlation function $R_{XX}(\tau)$

15. a(i) Show that if the input $\{X(t)\}$ is a WSS process for a linear system then output $\{Y(t)\}$ is a WSS process. Also find $R_{XY}(\tau)$

(ii) If $X(t)$ is the input voltage to a circuit and $Y(t)$ is the output voltage. $\{X(t)\}$ is a stationary random process with $\mu_X = 0$ and $R_{XX}(\tau) = e^{-2|\tau|}$. Find the mean μ_Y and power spectrum $S_{YY}(\omega)$ of the output if the system transfer function is given by $H(\omega) = \frac{1}{\omega + 2i}$

OR

b(i) A stationary random process $X(t)$ having the autocorrelation function $R_{XX}(\tau) = A f(\tau)$ is applied to a linear system at time $t = 0$ where $f(\tau)$ represents the impulse function. The linear system has the impulse response of $h(t) = e^{-bt} u(t)$ where $u(t)$ represents the unit step function. Find $R_{YY}(\tau)$. Also find the mean and variance of $Y(t)$.

(ii) If $N(t)$ is a band limited white noise centered at a carrier frequency ω_0 such that

$$S_{NN}(\omega) = \begin{cases} \frac{N_0}{2}, & |\omega - \omega_0| < \omega_B \\ 0, & \text{otherwise} \end{cases}$$

find the auto correlation of the process $\{N(t)\}$.