

AALIM MUHAMMED SALEGH COLLEGE OF ENGINEERING

Department of Mathematics

Model Examination

IV Semester B.E. ( ECE )

MA2261 –PROBABILITY AND RANDOM PROCESSES

Time : 3 hours

Max.Marks:100

**Part A (10 X 2=10 Marks)**

Answer ALL questions

1. Find the moment generating function of binomial distribution.
2. A continuous random variable X has probability density function  $f(x) = \begin{cases} 3x^2, & 0 \leq x \leq 1 \\ 0, & \text{other wise} \end{cases}$  Find k such that  $P(X > k) = 0.5$
3. Find the acute angle between the two lines of regression, assuming the two lines of regression.
4. State Central Limit Theorem for iid random variables.
5. Define a wide sense stationary process.
6. State the postulates of a Poisson process.
7. Prove that  $S_{xy}(\omega) = S_{yx}(-\omega)$
8. State any two properties of cross correlation function.
9. Define a linear time - invariant system.
10. Define Band-Limited white noise.

**PART-B (5 x 16 = 80 marks)**

11. a(i) The probability density function of a random variable X is given by  $f(x) = \begin{cases} x, & 0 < x < 1 \\ k(2-x), & 1 < x < 2 \\ 0, & \text{otherwise} \end{cases}$ 
  - (1) Find the value of k
  - (2) Find  $P(0.2 < x < 1.2)$
  - (3) What is  $P[0.5 < x < 1.5/x = 1]$
  - (4) Find the distribution function of f(x).(ii) Describe gamma distribution. Obtain its moment generating function. Hence compute its mean and variance.

OR

- b(i) An office has four phone lines. Each is busy about 10% of the time. Assume that the phone lines act of independently.
    - (1) What is the probability that all four phones are busy ?
    - (2) What is the probability that atleast two of them are busy ?
  - (ii) If X is uniformly distributed in (-1, 1), then find the probability density function of  $Y = \sin \frac{\pi X}{2}$
12. a(i) The joint pdf of a two-dimensional random variable (X,Y) is given by  $f(x,y) = \begin{cases} xy^2 + \frac{x^2}{8}; & 0 \leq x \leq 2, 0 \leq y \leq 1 \\ 0 & ; \text{otherwise} \end{cases}$  Compute (i)  $P(Y < \frac{1}{2})$  (ii)  $P(X > 1/Y < \frac{1}{2})$  (iii)  $P(X + Y \leq 1)$
  - (ii) Let  $X_1, X_2, X_3, \dots, X_n$  are uniform variates with mean =2.5 and variance = 3/4, use CLT to estimate  $P(108 \leq S_n \leq 126)$  where  $S_n = X_1 + X_2 + X_3 + \dots + X_n$  n = 48.

OR

- b(i) Let ( X Y, ) be a two dimensional random variable and the probability density function be given by  $f(x,y) = x+ y, 0 \leq x, y \leq 1$ , Find the p.d.f of  $U = XY$
  - (ii) The regression equation of X on Y is  $3Y - 5X + 108 = 0$ . If the mean value of Y is 44 and the variance of X is 9/16th of the variance of Y . Find the mean value of X and the correlation coefficient.
13. a(i) Define a semi random telegraph signal process and prove that it is evolutionary

(ii) A random process has sample functions of the form  $X(t) = A \cos(\omega t + \theta)$  where  $\omega$  is constant,  $A$  is a random variable with mean zero and variance one and  $\theta$  is a random variable that is uniformly distributed between  $0$  and  $2\pi$ . Assume that the random variables  $A$  and  $\theta$  are independent. Is  $X(t)$  a mean - ergodic process ?

OR

b(i) If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2 per minute, find the probability that the interval between 2 consecutive arrivals is

(1) more than 1 minute (2) between 1 minute and 2 minutes and (3) 4 min. or less.

(ii) Prove that the interval between two successive occurrences of a Poisson process with parameter  $\lambda$  has an exponential distribution with mean  $1/\lambda$ .

(iii) Prove that a random telegraph signal process  $Y(t) = \alpha X(t)$  is a Wide Sense Stationary Process when  $\alpha$  is a random variable which is independent of  $X(t)$ , assume value  $-1$  and  $+1$  with equal probability and

$$R_{XX}(t_1, t_2) = e^{-2\lambda|t_1 - t_2|}$$

14. a(i) Find the autocorrelation function of the periodic time function of the period time function  $\{X(t)\} = A \sin \omega t$

(ii) Find the power spectral density of the random process whose auto correlation function is

$$R(\tau) = \begin{cases} 1 - |\tau|, & \text{for } |\tau| \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

OR

b(i) The cross-power spectrum of real random processes  $\{X(t)\}$  and  $\{Y(t)\}$  is given by

$$S_{xy}(\omega) = \begin{cases} a + \frac{ib\omega}{\alpha}, & \text{for } -\alpha < \omega < \alpha \text{ and } \alpha > 0 \\ 0, & \text{elsewhere} \end{cases}$$

function.

(ii) If  $\{X(t)\}$  and  $\{Y(t)\}$  are two random processes with auto correlation function  $R_{XX}(\tau)$  and  $R_{YY}(\tau)$  respectively

then prove that  $|R_{XY}(\tau)| \leq \sqrt{R_{XX}(0)R_{YY}(0)}$  Establish any two properties of auto correlation function  $R_{XX}(\tau)$

15. a(i) Show that if the input  $\{X(t)\}$  is a WSS process for a linear system then output  $\{Y(t)\}$  is a WSS process. Also find  $R_{XY}(\tau)$

(ii) If  $X(t)$  is the input voltage to a circuit and  $Y(t)$  is the output voltage.  $\{X(t)\}$  is a stationary random process with  $\mu_X = 0$  and  $R_{XX}(\tau) = e^{-2|\tau|}$ . Find the mean  $\mu_Y$  and power spectrum  $S_{YY}(\omega)$  of the output if the system transfer function is given by  $H(\omega) = \frac{1}{\omega + 2i}$

OR

b(i) A stationary random process  $X(t)$  having the autocorrelation function  $R_{XX}(\tau) = A f(\tau)$  is applied to a linear system at time  $t = 0$  where  $f(\tau)$  represents the impulse function. The linear system has the impulse response of  $h(t) = e^{-bt} u(t)$  where  $u(t)$  represents the unit step function. Find  $R_{YY}(\tau)$ . Also find the mean and variance of  $Y(t)$ .

(ii) If  $N(t)$  is a band limited white noise centered at a carrier frequency  $\omega_0$  such that

$$S_{NN}(\omega) = \begin{cases} \frac{N_0}{2}, & |\omega - \omega_0| < \omega_B \\ 0, & \text{otherwise} \end{cases}$$

find the auto correlation of the process  $\{N(t)\}$ .