AALIM MUHAMMED SALEGH COLLEGE OF ENGINEERING

Department of Mathematics

Model Examination

IV Semester B.E. (ECE)

MA2261 –PROBABILITY AND RANDOM PROCESSES

Time : 3 hours Max.Marks:100

Part A (10 X 2=10 Marks)

Answer ALL questions

1. Find the moment generating function of binomial distribution.
2. A continuous random variable X has probability density function f(x)= $\begin{cases} 3x^2, & 0 \le x \le 1 \\ 0, & other\ wise \end{cases}$ Find k such that P(X > k) = 0.5
3. Find the acute angle between the two lines of regression, assuming the two lines of regression.
4. State Central Limit Theorem for iid random variables.
5. Define a wide sense stationary process.
6. State the postulates of a Poisson process.
7. Prove that $S_{xy}(\omega) = S_{yx}(-\omega)$
8. State any two properties of cross correlation function.
9. Define a linear time - invariant system.
10. Define Band-Limited white noise.

PART-B (5 x 16 = 80 marks)

11. a(i) The probability density function of a random variable X is given by $f(x) = \begin{cases} x, & 0 < x < 1 \\ k(2-x), & 1 < x < 2 \\ 0, & otherwise \end{cases}$

(1)Find the value of k (2) Find P(0.2 < x < 1.2) (3) What is P[0.5 < x < 1.5/x = 1]
(4) Find the distribution function of f(x).

(ii) Describe gamma distribution. Obtain its moment generating function. Hence compute its mean and variance.

OR

b(i) An office has four phone lines. Each is busy about 10% of the time. Assume that the phone lines act of independently. (1) What is the probability that all four phones are busy ?
(2) What is the probability that atleast two of them are busy ?

(ii) If X is uniformly distributed in (-1, 1), then find the probability density function of Y = $\sin\frac{\pi X}{2}$

12. a(i)The joint pdf of a two-dimensional random variable (X,Y) is given by

$f(x,y) = \begin{cases} xy^2 + \frac{x^2}{8} & ; 0 \le x \le 2,\ 0 \le y \le 1 \\ 0 & ; otherwise \end{cases}$ Compute (i) $P(Y < \frac{1}{2})$ (ii) $P(X > 1 / Y < \frac{1}{2})$ (iii) $P(X + Y \le 1)$

(ii) Let X_1 , X_2 , X_3,..., X_n are uniform variates with mean =2.5 and variance = 3/4, use CLT to estimate $P(108 \le Sn \le 126]$ where $Sn = X_1 + X_2 + X_3 + ... + X_n$ n = 48.

OR

b(i) Let (X Y,) be a two dimensional random variable and the probability density function be given by f(x,y) = x+ y, 0≤ x , y ≤ 1, Find the p.d.f of U = XY

(ii) The regression equation of X on Y is 3Y – 5X + 108 = 0. If the mean value of Y is 44 and the variance of X is 9/16th of the variance of Y . Find the mean value of X and the correlation coefficient.

13. a(i) Define a semi random telegraph signal process and prove that it is evolutionary

(ii) A random process has sample functions of the form $X(t) = A\cos(\omega t + \theta)$ where ω is constant, A is a random variable with mean zero and variance one and θ is a random variable that is uniformly distributed between 0 and 2π. Assume that the random variables A and θ are independent. Is X(t) is a mean - ergodic process ?

OR

b(i) If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2 per minute, find the probability that the interval between 2 consecutive arrivals is

(1) more that 1 minute (2) between 1 minute and 2 minute and (3) 4 min. or less.

(ii) Prove that the interval between two successive occurrences of a Poisson process with parameter λ has an exponential distribution with mean $1/\lambda$.

(iii) Prove that a random telegraph signal process $Y(t) = \alpha X(t)$ is a Wide Sense Stationary Process when α is a random variable which is independent of X(t), assume value -1 and +1 with equal probability and

$R_{XX}(t_1, t_2) = e^{-2\lambda|t1-t2|}$

14. a(i) Find the autocorrelation function of the periodic time function of the period time function $\{X(t)\} = A\sin \omega t$

(ii) Find the power spectral density of the random process whose auto correlation function is

$$R(\tau) = \begin{cases} 1-|\tau|, & \text{for } |\tau| \le 1 \\ 0 & \text{otherwise} \end{cases}$$

OR

b(i) The cross-power spectrum of real random processes {X(t)} and {Y(t)} is given by

$S_{xy}(\omega) = \begin{cases} a + \frac{ib\omega}{\alpha}, for -\alpha < \omega < \alpha \text{ and } \alpha > 0 \\ 0, \text{ elsewhere} \end{cases}$ where a and b are constants. Find the cross correlation function.

(ii) If {X(t)} and {Y (t)} are two random processes with auto correlation function $R_{XX}(\tau)$ and $R_{YY}(\tau)$ respectively

then prove that $|R_{XY}(\tau)| \le \sqrt{R_{XX}(0)R_{YY}(0)}$ Establish any two properties of auto correlation function $R_{XX}(\tau)$

15. a(i) Show that if the input {X(t)} is a WSS process for a linear system then output {Y (t)} is a WSS process. Also find $R_{XY}(\tau)$

(ii) If X(t) is the input voltage to a circuit and Y (t) is the output voltage. {X(t)} is a stationary random process with $\mu_X = 0$ and $R_{XX}(\tau) = e^{-2|\tau|}$. Find the mean μ_Y and power spectrum $S_{YY}(\omega)$ of the output if the system transfer function is given by $H(\omega) = \frac{1}{\omega+2i}$

OR

b(i) A stationary random process X(t) having the autocorrelation function $R_{XX}(\tau) = A\,\delta(\tau)$ is applied to a linear system at time t = 0 where $f(\tau)$ represents the impulse function. The linear system has the impulse response of $h(t) = e^{-bt}u(t)$ where u(t) represents the unit step function. Find $R_{YY}(\tau)$. Also find the mean and variance of Y (t).

(ii) If N(t) is a band limited white noise centered at a carrier frequency ω_0 such that

$S_{NN}(\omega)=\begin{cases} \frac{N_0}{2}, & |\omega - \omega_0| < \omega_B \\ 0, & otherwise \end{cases}$ find the auto correlation of the process{N(t)}.